

Anderson Localization in Atoms

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We investigate the celebrated analogy [1] between Anderson localization phenomena in condensed matter physics [2] and related effects in the ionization process of driven Rydberg atoms, and put them on a quantitative footing.

The atomic system we consider for a comprehensive numerical analysis of its ionization properties is a highly excited hydrogen atom exposed to a monochromatic electromagnetic field of linear polarization. The external field is purely classical, we ignore relativistic corrections, and assume an infinite proton mass. Dropping the – for our purposes negligible – ponderomotive energy shift [3], and restricting to the dipole approximation, the Hamiltonian (in atomic units, and applying the velocity gauge [3]) reads as

$$H = \frac{1}{2}\vec{p}^2 - \frac{1}{r} - \frac{Fp_z}{\omega} \sin(\omega t). \quad (1)$$

Our method of a combined application of the Floquet theorem and of complex scaling [4] allows an exact description of the quantum dynamics of the hydrogen atom, and fully includes the Coulomb singularity, as well as the field-induced coupling to the atomic continuum. Our only restriction here is a confinement of the electron to a one-dimensional configuration space along the polarization axis of the external field. The approximation of reduced dimensionality is reasonable for quasi-one-dimensional Rydberg states for which the electronic motion is essentially confined along the polarization axis [5].

The coupling to the atomic continuum induced by the external field forces all bound states to turn into resonances. These resonances are eigensolutions of the associated scattering problem, which satisfies purely outgoing (Siegert-)boundary conditions [6], and they are characterized by quasi-energies ϵ with a resonance width Γ_ϵ . In our approach, these quasi-energies are obtained by the method of complex scaling, and are given by the complex eigenvalues $\epsilon = \text{Re}(\epsilon) - i\Gamma_\epsilon/2$ of the scaled Floquet Hamiltonian. The ionization probability of the initial – field free – Rydberg state $|n_0\rangle$ with principal quantum number n_0 is completely defined by the ionization widths (or rates) Γ_ϵ of the individual Floquet eigenstates of the atom in the field. Assuming rapid switching of the microwave field, the weights w_ϵ of the individual Floquet states are simply given by their time-averaged overlaps with $|n_0\rangle$. Consequently, the ionization probability reads as [4]

$$\begin{aligned} P_{\text{Ion}}(t) &= 1 - P_{\text{bound}}(t) \\ &= 1 - \sum_{\epsilon} e^{-\Gamma_{\epsilon} t w_{\epsilon}}, \end{aligned} \quad (2)$$

where t is the total interaction time of the atom with the driving field. The sum in (2) runs over the entire Floquet spectrum.

In the same way as the electronic transport through a disordered solid is described by its transmission properties or the conductance [2], respectively, we define an atomic conductance g [7] to allow for a statistical characterization of the energy transport in periodically driven atoms. g is given by the weighted sum of the ionization rates Γ_ϵ , which represent the atomic analogue of the transmission rates through a disordered solid-state sample. From (2), it follows that

$$g \equiv \frac{1}{\Delta} \sum_{\epsilon} \Gamma_{\epsilon} w_{\epsilon} = \frac{1}{\Delta} \frac{d}{dt} \Big|_{t \approx 0} P_{\text{Ion}}(t), \quad (3)$$

with Δ being the mean level spacing of the Floquet spectrum. An estimate for Δ is obtained [7] by

$$\Delta \equiv \frac{\omega}{N}, \quad (4)$$

where $N \equiv 1/(2n_0^2\omega)$ counts the number of photons the atom must absorb in order to ionize.

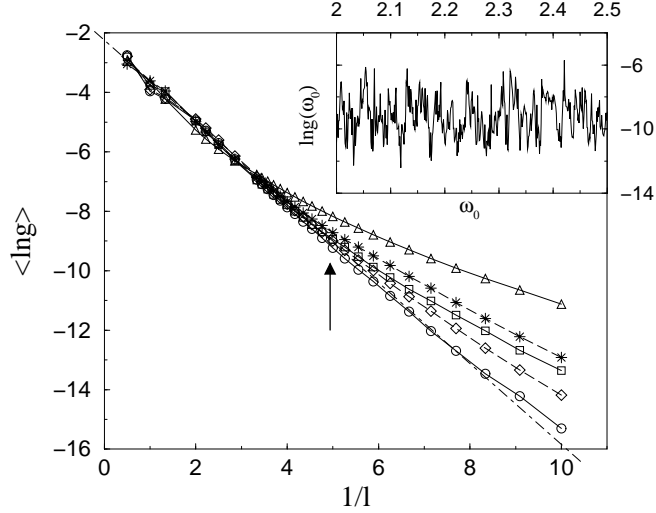


Figure 1: Averaged values of the natural logarithm of the atomic conductance g vs. the inverse localization ratio $1/\ell$ (initial states $n_0 = 40$ (triangles), 60 (stars), 70 (squares), 90 (diamonds), 100 (circles), $\ell = 0.1 - 2$). Each data point is obtained by averaging over an ensemble of 500 different samples (with the frequency measured in units of the classical, undisturbed Kepler frequency $\omega_0 \equiv \omega \times n_0^3 \in [2.0; 2.5]$). For the initial principal quantum number $n_0 = 100$ we observe an almost linear dependence along the inserted straight line (dashed-dotted), a result which is also obtained for disordered solid-state models in the localized regime [8]. There is a clear trend to approach the linear behaviour – which one expects from the theory of dynamical localization – as n_0 increases from 40 to 100. The inset shows the huge fluctuations of g – on a logarithmic scale! – as a function of the external (scaled) frequency, for $\ell = 0.2$ and $n_0 = 100$ (corresponding to the data point marked by the arrow in the main figure).

Despite the fully deterministic dynamics which are generated by (1), the widths Γ_ϵ and the overlaps w_ϵ are very sensitive to slight changes of the field parameters. This behaviour forces the ionization probability and the conductance, respectively, to strongly fluctuate as we vary, for instance, the microwave frequency. The observed huge fluctuations (cf. inset in fig. 1) are the analogue of conductance fluctuations known from condensed matter physics [2], where disorder induced interference effects lead to an exponential localization of the electronic wave function along the lattice, and therefore to a strong reduction of the transmission through the solid. In both, the atomic and the solid-state problem, a statistical description is needed for characterizing this complex transport behaviour. Extreme fluctuations typically occur in a regime where the overall – atomic or mesoscopic – conductance is exponentially small. Then, the theory of dynamical localization [9], which was developed in close analogy to the concept of Anderson localization in condensed matter physics, predicts an exponential decrease of the bound-state population distribution towards the atomic continuum threshold. This decay is completely characterized by the ratio of the localization length ξ (which measures the width of the bound-state distribution) to N . We quantitatively check this prediction of dynamical localization by plotting the logarithmic atomic conductance $\ln g$ vs. the inverse ratio $\ell^{-1} \equiv N/\xi$. For very high initial Rydberg states, with principal quantum numbers close to 100, we observe an exponential decrease of the atomic conductance (cf. fig. 1). However, for smaller n_0 , our results clearly show systematic deviations from the exponential behaviour. The same deviations are observable by looking at the statistical properties of the atomic conductance [10]. Keeping the ratio ℓ fixed and changing the frequency of the externally applied field in 500 equidistant, small steps, we obtain an ensemble of 500 different “samples” for each value of n_0 and ℓ . Fig. 2 shows the log-normal distributions of the atomic conductance, which is in good agreement with the predictions of solid-state models [11]. Again, the deviation of the distributions are the larger from the expected log-normal behaviour, the smaller we have chosen the initial principal quantum number n_0 .

The crucial insight gained from our study of the statistical properties of the atomic conductance g is that the initial Rydberg state defines an additional parameter $n_0 \sim \hbar_{\text{eff}}^{-1}$ [12] (as compared to the scenario in Anderson-localized solids) which non-trivially affects the statistical behaviour

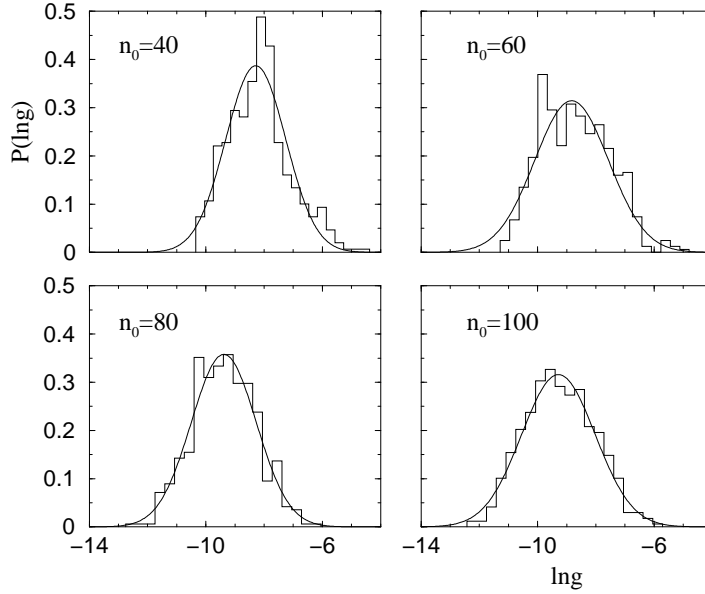


Figure 2: Direct comparison of the statistical distributions of $\ln g$ (histograms) and the corresponding Gaussian fits (thick lines), for the initial states $n_0 = 40, 60, 80, 100$ (localization ratio $\ell = 0.2$). As the principal quantum number n_0 increases at constant ℓ , a manifest improvement of the approximation by the fits, and a trend to shift to lower mean values is visible. Similar results are obtained for the entire localized region $\ell < 0.5$.

of the atomic transport process. Furthermore, the observed fluctuations prove to be quite robust with respect to slight uncertainties in the applied field strengths (the main problem in laboratory experiments [13]), calling for a direct experimental verifications of our numerical results.

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